JSL

2011

MATHEMATICS (Optional)
Paper – II

340041

Standard : Degree | **Total Marks : 200**
Nature : Conventional (Essay) type | **Duration : Three hours**

N.B. : *1) Answers must be written in English.*

*2) Question **No. 1** is **compulsory**. Of the remaining questions, attempt **any Four** selecting one question from **each section**.*

*3) Figures to the **RIGHT** indicate marks of the respective question.*

4) Use of log table, Non-Programmable calculator is permitted, but any other Table/Code/Reference book are not permitted.

5) Number of optional questions upto the prescribed number in the order in which they have been solved will only be assessed. Excess answers will not be assessed.

6) Credit will be given for orderly, concise and effective writing/presentation.

7) Candidates should not write roll number, any name (including their own), signature, address or any indication of their identity anywhere inside the answer book otherwise he/she will be penalised.

SEAL

Marks

Answer **any four** of the following (**10 marks each**) : **(40 Marks)**

1. (a) Show that S_4, the permutation group over 4 symbols has no element of order 6. **10**

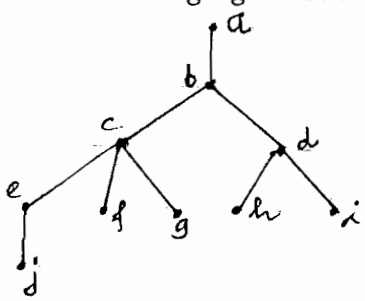
(b) State Cauchy residue theorem and evaluate $\int_{-\infty}^{\infty} \frac{dx}{x^4+1}$. **10**

(c) The ends of a laterally insulated rod of length 2 are kept at 0° C while the initial temperature at interior points u(x, 0) is given by **10**

$$u(x, 0) = \sin \frac{\pi x}{2} + 3 \sin 5\frac{\pi x}{2}, \; 0 < x < 2.$$

Determine the temperature in the bar at a distance x from one and at any subsequent time t ; u(x, t).

(d) The following figure shows the tree with root at a **10**



P.T.O.

Marks

Determine :

(i) parents of c and h

ii) children of d and e

(iii) descendents of c and e

(iv) siblings of f and h

(v) leaves.

(e) Show that every infinite bounded set of real numbers has at least one limit point. 10

SECTION – A

2. (a) Show that every homomorphic image of a group is isomorphic to some quotient group of G. 15

(b) Define principal ideal ring. Show that a polynomial ring over a field is a principal ideal ring. 15

(c) If K is an extension field of F, $a \in K$ is algebraic over F and p(x) is a minimal polynomial of a over F, then show that p(x) is irreducible over F. 10

3. (a) Show that quotient group of an abelian group is abelian but the converse is not true. 15

(b) Define Euclidean ring. Show that the ring of Gaussian integers is a Euclidean ring. 15

(c) If Q is the field of rational numbers, then show that $Q\left(\sqrt{2},\sqrt{3}\right)=Q\left(\sqrt{2}+\sqrt{3}\right)$. 10

SECTION – B

4. (a) Suppose S is an ordered set with least upper bound property, $B \subset S$, B is not empty and B is bounded below. Let L be the set of all lower bounds of B. Then show that $\alpha = \sup L$ exists in S and $\alpha = \inf B$. 15

(b) Let $\Sigma\ a_n$ be a series of real numbers which converges but not absolutely. Show that for any real α, there is a rearrangement $\Sigma\ a'_n$ with partial sums s'_n such that $s'_n \to \alpha$. 15

(c) Show that the Cauchy Riemann equations are necessary conditions for a given function w = f(z) = u + iv defined in a region of complex plane to be analytic. 10

5. (a) Show that every continuous function from a compact metric space X into a metric space Y is uniformly continuous. 15

(b) If $\{f_n\}$ is a sequence of continuous functions on $E \subset \mathbb{R}$ and if $f_n \to f$ uniformly on E, then show that f is continuous on E. 15

Marks

(c) If f(z) is analytic in a simply connected domain D bounded by contour C, then show that for any z_0 in the interior of D, the following holds 10

$$f(z_0) = \frac{1}{2\pi i} \oint_C \frac{f(z)}{z - z_0} dz$$

Marks

SECTION – C

6. (a) Determine the general solution of the differential equation 15

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = e^x \cos 3x.$$

(b) Compute $\frac{\partial z}{\partial x}$ and $\frac{\partial y}{\partial x}$ at (1, – 1, 2) if $x^2 + y^2 + z^2 = 25$. 10

(c) Solve the partial differential equation 15

$$(y^2 + z^2)\frac{\partial z}{\partial x} - xy\frac{\partial z}{\partial y} + zx = 0.$$

7. (a) Determine the general solution of the differential equation 15

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + y = \sin^2 x.$$

(b) Compute $\frac{d^2y}{dx^2}$ if $x^5 + y^5 - 5x^2 = 0$. 10

(c) Solve the partial differential equation 15

$$(x^2 - y^2 - yz)\frac{\partial z}{\partial x} + (x^2 - y^2 - zx)\frac{\partial z}{\partial y} = z(x - y).$$

SECTION – D

8. (a) Use Newton-Raphson's method to determine the root of the equation $x^4 + x^3 - 7x^2 - x + 5 = 0$ which lies between 2 and 3 correct upto three decimal places. 10

(b) Use Gauss-Seidel method to solve the following system of equations 10

$$2x_1 - x_2 + 2x_3 = 3$$
$$x_1 + 3x_2 + 3x_3 = -1$$
$$x_1 + 2x_2 + 5x_3 = 1$$

Select $x_1 = 0.3$, $x_2 = -0.8$ and $x_3 = 0.3$ as the initial approximation.

Marks

(c) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by using Simpson' s one-third rule. Use it to obtain the approximate value of π and find error in this calculated value. Select $h = \frac{1}{6}$. **10**

(d) Use simplex method to solve the following linear programming problem Maximize $Z = 12x_1 + 15x_2 + 14x_3$, subject to the constraints **10**

$$-x_1 + x_2 \le 0$$
$$-x_2 + 2x_3 \le 0$$
$$x_1 + x_2 + x_3 \le 100$$
$$\text{and } x_1, x_2, x_3, \ge 0.$$

9. (a) Determine the approximate value, correct to three places of decimals for the equation $x^3 - 3x + 4 = 0$ by using the method of false position. Given that the root lies between -2 and -3. **10**

(b) Solve the following system of equations : **10**

$$6x_1 + x_2 + x_3 = 105$$
$$4x_1 + 8x_2 + 3x_3 = 155$$
$$5x_1 + 4x_2 - 10x_3 = 65$$

by using Gauss-Seidel method. Select $x_1 = x_2 = x_3 = 0$ as the initial appoximation.

(c) Evaluate $\int_2^{10} \frac{dx}{1+x}$ by using Simpson's one-third rule. Divide (2, 10) into eight equal parts. Use it to find the approximate value of $\log\left(\frac{11}{3}\right)$. Also, find the error in the value thus calculated. **10**

(d) Use Simplex method to solve the following linear programming problem : Maximize $Z = 4x_1 + 3x_2 + 6x_3$, subject to the constraints **10**

$$2x_1 + 3x_2 + 2x_3 \le 440$$
$$4x_1 + 3x_3 \le 470$$
$$2x_1 + 5x_2 \le 430$$
$$\text{and } x_1, x_2, x_3 \ge 0.$$